



Non-ideal Stirling engine thermodynamic model suitable for the integration into overall energy systems



Joseph A. Araoz^{a, b, *}, Marianne Salomon^a, Lucio Alejo^b, Torsten H. Fransson^a

^a Department of Energy Technology, School of Industrial Technology and Management (ITM), Royal Institute of Technology (KTH), 100 44 Stockholm, Sweden

^b Facultad de Ciencias y Tecnología (FCyT), Universidad Mayor de San Simón (UMSS), Cochabamba, Bolivia

HIGHLIGHTS

- A numerical thermodynamic model for Stirling engine systems was developed.
- Thermodynamic equations were coupled with the heat transfer governing equations.
- The model was validated with experimental and numerical data.
- The brake power and engine efficiency at different conditions were calculated.
- Additional model results provide a deeper insight into the engine operation.

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ABSTRACT

The reliability of modelling and simulation of energy systems strongly depends on the prediction accuracy of each system component. This is the case of Stirling engine-based systems, where an accurate modelling of the engine performance is very important to understand the overall system behaviour. In this sense, many Stirling engine analyses with different approaches have been already developed. However, there is a lack of Stirling engine models suitable for the integration into overall system simulations. In this context, this paper aims to develop a rigorous Stirling engine model that could be easily integrated into combined heat and power schemes for the overall techno-economic analysis of these systems. The model developed considers a Stirling engine with adiabatic working spaces, isothermal heat exchangers, dead volumes, and imperfect regeneration. Additionally, it considers mechanical pumping losses due to friction, limited heat transfer and thermal losses on the heat exchangers. The model is suitable for different engine configurations (alpha beta and gamma engines). It was developed using Aspen Custom Modeller[®] (ACM[®]) as modelling software. The set of equations were solved using ACM[®] equation solver for steady-state operation. However, due to the dynamic behaviour of the cycle, a C++ code was integrated to solve iteratively a set of differential equations. This resulted in a cyclic steady-state model that calculates the power output and thermal requirements of the system. The predicted efficiency and power output were compared with the numerical model and the experimental work reported by the NASA Lewis Research Centre for the GPU-3 Stirling engine. This showed average absolute errors around $\pm 4\%$ for the brake power, and $\pm 5\%$ for the brake efficiency at different frequencies. However, the model also showed large errors ($\pm 15\%$) for these calculations at higher frequencies and low pressures. Additional results include the calculation of the cyclic expansion and compression work; the pressure drop and heat flow through the heat exchangers; the conductive, shuttle effect and regenerator thermal losses; the temperature and mass flow distribution along the system; and the power output and efficiency of the engine. These results show that the model allows an extensive study of different parameters of the engine and thus it is suitable for design optimization studies. In addition, it also presents the capability for the integration into overall Stirling engine combined heat and power systems and therefore will allow the performance evaluation of the engine integrated on these systems.

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* Corresponding author. Department of Energy Technology, School of Industrial Technology and Management (ITM), Royal Institute of Technology (KTH), 100 44 Stockholm, Sweden. Tel.: +46 720117549; fax: +46 08 790 7477.

E-mail addresses: adhemararaoz@gmail.com, araoz@kth.se (J.A. Araoz).

Nomenclature

A	area (m ²)
A _o	external wet area of the tube (m ²)
C _f	non-dimensional friction coefficient
C _{fd}	form drag coefficient
C _{sf}	skin friction coefficient
C _p	constant pressure specific heat (J/kg K)
C _{pwater}	constant pressure specific heat for inlet water (J/kg K)
D _d	displacer diameter (m)
d _i	internal diameter of the tube (m)
d _{hy}	hydraulic diameter (m)
Err	error tolerance
Error1	absolute error calculated for T _c and T _e
Error2	absolute error calculated for T _k and T _h
Error3	absolute error calculated for T _{wh} and T _{wk}
f	friction factor coefficient
freq	engine frequency (1/s)
F _R	view factor
h	convective heat transfer coefficient (W/m ² K)
h _r	radiation heat transfer coefficient (W/m ² K)
h _{water}	water film heat transfer coefficient (W/m ² K)
J	annular gap cylinder displacer (m)
k	thermal conductivity (W/m K)
L	tube length (m)
m	mass of the fluid (kg)
M	molecular weight (kg/mol)
m _{water}	mass flow of the inlet water (kg/s)
NTU	number of transfer units
Nu	non-dimensional Nusselt number
P	pressure (Pa)
P _{br}	engine brake power (W)
Pr	non dimensional Prandtl number
Q	heat transfer rate (W)
Q _{hc}	heater heat transfer rate by cycle (J/cycle)
Q _{kc}	cooler heat transfer rate by cycle (J/cycle)
Q _{rc}	regenerator heat transfer rate by cycle (J/cycle)
Q _{ht}	total heating requirement for the engine (W)
Q _{kt}	total cooling requirement for the engine (W)
Q _{lossr}	heat loss due to imperfect regenerator (W)
Q _{lk}	heat loss due to internal conduction (W)
Q _{lsh}	heat loss due to shuttle conduction (W)
R	gas constant (J/kg K)
Re	non-dimensional Reynolds number
R _{ci}	conductive thermal resistance for tubes wall (K/W)
R _{fi}	fouling thermal resistance inside the tubes (K/W)
R _{fo}	fouling thermal resistance outside the tubes (K/W)
R _{hi}	convective thermal resistance inside the tubes (K/W)

St	non-dimensional Stanton number
SE	Stirling engine
T	temperature (K)
T _{ad}	adiabatic flame temperature of the fuel (K)
T _{wi}	temperature at the internal wall of the tubes (K)
T _{wo}	temperature at the outer wall of the tubes (K)
T _{water_in}	inlet temperature of the water (K)
u	mean velocity (m/s)
v	mean velocity (m/s)
V	volume (m ³)
W _c	compression work (J/cycle)
W _e	expansion work (J/cycle)
W _{el}	electric work output (W)
W _n	net work by cycle (J/cycle)
W _{ploss}	energy loss due to pressure drop (J/cycle)
W _{br}	engine net brake work (J/cycle)
ΔP	pressure drop (N/m ²)
Z	displacer stroke (m)

Acronyms

ACM	Aspen custom modeller
CHP	Combined heat and power
LeRC	Lewis Research Center

Subscripts

c	compression space
d	displacer
k	cooler space
e	expansion space
f	final value
h	heater space
r	regenerator space
i	inside section
o	outside section
w	wall
wg	wetted gas
pist	piston
0	initial value

Greek symbols

α	surface emissivity
γ	adiabatic constant
η _b	brake efficiency
σ	Stefan–Boltzmann constant (W/m ² K ⁴)
ε	regenerator effectiveness
ρ	fluid density (kg/m ³)
θ	crank angle (rad)
μ	viscosity (kg/m s)

1. Introduction

Increasing energy demands, strict environmental regulations and availability of different renewable resources requires the development of environmentally friendly energy systems. This needs an intensive research for the development of efficient and sustainable power technologies. In this scenario, fuel flexible and energy efficient technologies, such as the Stirling engine (SE), are becoming renewed solutions to address these requirements.

Research on these engines was extensive since its invention but the fast growth of parallel technologies, such as internal

combustion engines, slowed its progress. However, the development of new materials combined with engineering advances and the potential advantages that the Stirling engine offers in terms of low emissions, low noise and fuel flexibility, has renewed the interest on its development [1–3]. In addition, the successfully integration with micro and small scale combined heat and power (CHP) systems [3–6] increased its potential to become a strong technological alternative. However, Stirling-CHP technology must overcome practical limitations, such as the lower electrical performance compared with other technologies [7]. These limitations arise from the complexities of the system, which must considers

the engine mechanical arrangement, the heat transfer through the different components, the fluid dynamics of the working gas in the engine and an adequate power control system. Consequently, the development of CHP systems needs a complete analysis of the integrated engine performance considering different geometrical and thermodynamic parameters. Therefore, the combination of experimental and simulation tests are essential for its design, evaluation and optimization. For this reason, the development of a model tool that simulates in detail the Stirling engine performance and could be easily integrated into combined heat and power schemes for an overall analysis of these systems is proposed in this paper.

The ideal cycle analysis is the simplest SE thermodynamic model. However, real performances largely differ from this ideal calculation. Considering this, different mathematical models have been developed to predict actual engine performances. According to Dyson et al. [8], these models could be categorized into zero order, approximate (first order), decoupled (second order), nodal analysis (third order), and the recently multidimensional analysis (fourth order).

Zero order methods estimate the power output considering the overall size of the engine. Beale [9] presented a correlation that estimates the power output of the engine and his approach was later completed by Senft [10], who derived the so called generalized Beale number. These methods are empirical and thus predict inaccurate power outputs, only suitable for engines under certain geometrical and operational conditions. First order methods estimate the power output and the work and heat flow during the engine cycle. These are based on Schmidt [8] analysis, who obtained closed-form solutions for the special case of sinusoidal volume variations and isothermal hot and cold spaces. Consequently the model accuracy is increased, but the isothermal assumption is still an idealized case. The modelling accuracy is improved with the second order or decoupled methods, which begin with a simplified cycle analysis for the estimation of the cyclic heat and work. Then, the power and heat losses, which are assumed independent, are evaluated and subtracted from this first approximation. Recent works such as Formosa [11], Cheng and Yu [12], Mehdizadeh and Stouffs [13], Urielli [14], and Parlak et al. [15] are based on this decoupled approach. This approach presents a reasonable accuracy with a fast solution, and thus became ideal for first design stages and optimization studies. On the other hand, third order nodal techniques increased the level of complexity and the accuracy in the models. These methods divide the engine into a network of nodes, and setup differential equations based on the conservation of mass, momentum and energy. Therefore, the decoupled assumption is replaced by considering the interaction between the different energy losses. Some of the analyses at this level were developed by Finkelstein [16], Urielli [17], Schock [18], Gedeon [19] and Tew et al. [20]. In addition Organ [21] and Larson [22], simplified the model solution using the method of characteristics for the case of unsteady one-dimensional flow. These methods present a detailed description of the engine performance, considering the heat and work flows and the thermal and mechanical losses. However, the computational time is largely increased and thus it is not suitable for optimization studies. Finally, fourth order methods, or multidimensional analysis has been recently developed thanks to the use of computer fluid dynamic (CFD) simulation codes. These multi-dimensional models provide detailed information regarding the flow pattern, temperature, and pressure distributions inside the engine. But, it requires much more computation efforts which are expensive and restricted to detailed engine design. This approach was used by Mahkamov [23], Ibrahim [24] and Wilson et al. [25].

The modelling techniques that were previously described are focused on stand-alone engine analysis and the suitability for the

integration into overall systems is not considered. Few works studied in detail this integration by simulation techniques [26–30]. These works are mainly based on the Annex 42 empirical model [31,32], which was developed for a particular engine and thus needs experimental data to adjust the results for other engine configurations. For this reason, this approach is closer to a “black box model” that does not reflect the thermodynamics of the engine. Therefore, the objective of this work is to develop an alternative Stirling engine model, which describes the thermodynamics, heat transfer and fluid dynamics through the engine. In addition, the modular approach considered smoothes the integration within overall CHP schemes and thus it is suitable for different engine configurations. Thereby, this model reflects the engine thermodynamics and it is suitable for overall CHP technoeconomic analysis.

Aspen Custom Modeller (ACM) was chosen as modelling tool due to its modular flow sheeting environment, algebraic differential equations solver, dynamic systems simulation and the possibility to link with external code sources (C++). The model proposed is based on the work of Urielli [14], assuming adiabatic working spaces and isothermal heat exchangers. The energy losses due dead volumes, pressure drop, limited heat transfer and imperfect regeneration are included. Additionally, external heat balances for the engine integration with the heat sources in the hot and cold side of the heat exchangers is considered. This model is applied for the simulation of the GPU-3 Stirling engine integrated within the external burner and a water cooling system as described on LeRC reports [33].

2. Second order model numerical formulation

The model for the Stirling engine system consists of four main modules named ideal adiabatic, internal heat transfer, external heat transfer, and energy losses. The system scheme and the relationship between the modules are shown in Fig. 1.

The first module represents the second order Stirling engine model developed by Urielli [14]. This model assumes ideal adiabatic compression and expansion spaces to estimate the main engine energy variables. The outputs are coupled to the internal heat transfer module, which through appropriate correlations evaluates the heat transfer and the temperature of the working fluid inside the heat exchangers. Fig. 1 shows the relation between both modules and the loop represents the iterative calculation of the working fluid temperature.

The next module includes the heat transfer between the external walls, considering the heat and cold sources. This is done through energy balances and heat transfer correlations, similarly to Hsu's approach [34]. There is also an iteration loop because the calculated external wall temperatures were approximated inputs for the adiabatic module. Finally, the last module includes energy losses to achieve results closer to the real engine performance.

The main variables that connect the modules are described below. Additionally, the block diagram shown in Fig. 1 is expanded into the scheme in Fig. 2, to represent these variables. This reflects the modular approach proposed to model the system:

- **External heat transfer module.** This module considers the adiabatic flame temperature and the inlet temperature of the cooling fluid on the hot and cold side respectively. Therefore, the heat source (Q_h) and the heat sink (Q_k) are used to estimate the wall temperatures (T_{woh} , T_{wok}). This approach is proposed to couple the Stirling engine within the external heat and cooling sources respectively.
- **Internal Heat Transfer module.** The internal working gas temperatures (T_h , T_k) in the heater and cooler respectively are

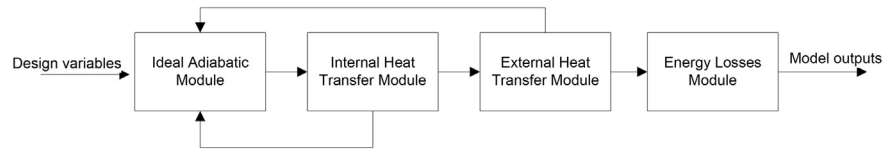


Fig. 1. Block diagram for the Stirling engine system.

calculated using heat transfer correlations for steady-state internal forced convective flow [35]. On the other hand, the regenerator analysis proposes the use of cyclic flow heat transfer correlations, which are more suitable for the flow conditions on this space [36]. Therefore, with these correlations the effect of limited heat transfer inside the engine is introduced on the model.

- **Ideal Adiabatic module.** The main operative variables such as electrical output (W_{el}), heat and cooling demands (Q_h , Q_k), are calculated considering the internal working fluid, temperature distribution, and the engine geometric characteristics following Urielli [14] approach.
- **Energy losses Module.** The losses inside the engine are estimated to correct the ideal adiabatic outputs. This module considers the losses due to pressure drop, axial conduction, shuttle heat transfer, and imperfect regeneration.

2.1. Stirling engine ideal adiabatic module

The model is based on Urielli [14] assumptions. The engine was divided into five different cells or control volumes that correspond to the expansion, heater, cooler, regenerator and the compression spaces. Additionally, each cell is described by its corresponding state variables (volume, mass, temperature and pressure), and flow variables (heat, work and mass flow).

The set of equations for the adiabatic model is obtained through corresponding mass and energy balances applied to each control volume. The complete set of algebraic differential equations is given by Urielli [14] and also in Appendix A. This set consists of twenty two variables and sixteen derivatives to be solved over a complete cycle. Considering this, Aspen Custom Modeller was coupled with an external C++ code, to solve the problem iteratively until cyclic steady-state conditions are reached.

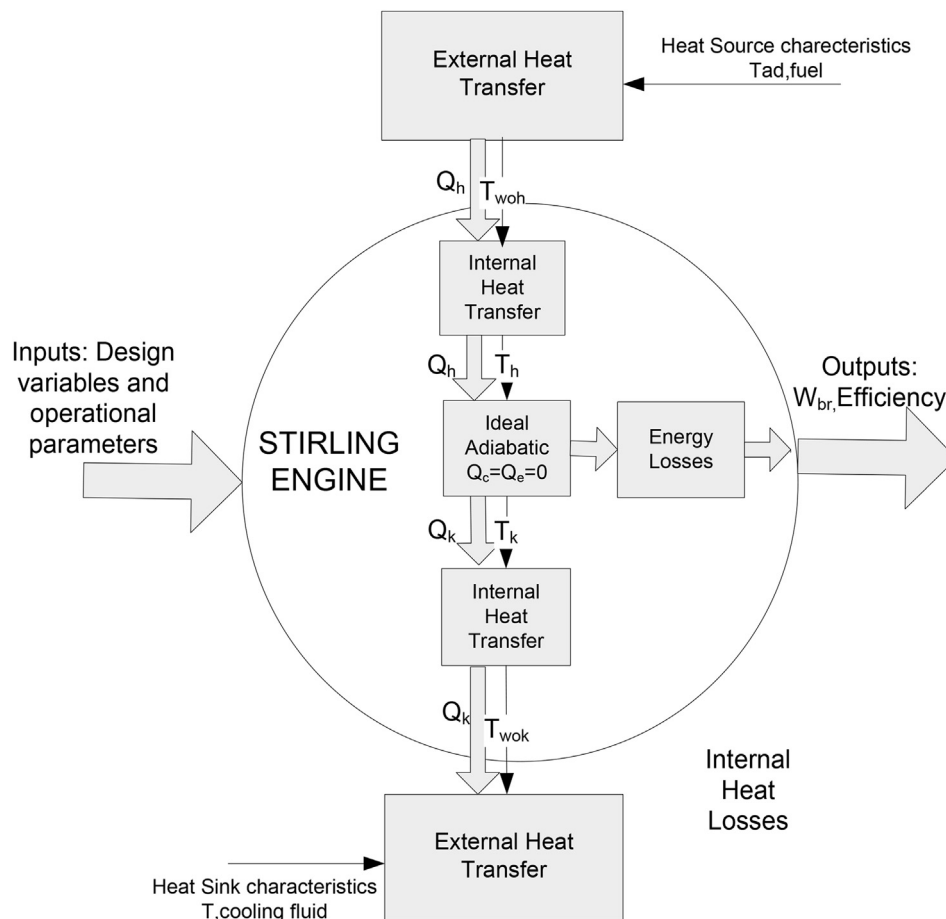


Fig. 2. Scheme of the Stirling engine model.

2.2. Heat transfer model

The heat transfer model considers the thermal energy that flows in and out of the engine. The heat flow is transferred to the working fluid via the heat exchangers at the hot and cold sides. This is shown in Fig. 3.

The heat transfer model is divided into:

- The external heat transfer module, which considers the heat flow from the hot or cold source to the external walls of the heat exchangers. This model considers that the main heat transfer mechanism from the hot source to the heater wall is by radiation. This assumption was probe feasible at higher temperatures, as it is shown in the work of Hsu et al. [37].
- The internal heat transfer module which considers the heat flow from the heat exchanger external walls, to the working fluid inside the heat exchanger

2.2.1. Internal heat transfer module

The ideal heat transfer assumption in the adiabatic model, implied an infinite heat transfer coefficient (h_i), and consequently the temperature of the heat exchangers walls were equal to the working gas temperatures (T_h , T_k). However, this assumption is very ideal and the heat transfer characteristics considering conduction, convection and fouling in both the heater and cooler heat exchangers are included in this module.

2.2.1.1. Heater. The heat transfer phenomenon from the heater wall to the working gas is considered as: Conductive heat transfer through the outer to the inner wall surface, and convective heat transfer from the inner wall surface to the working gas. Additionally, according to Hsu [37], it is considered that the heat flux is constant, normal to the surface and the radiation and axial heat transfer mechanisms are neglected.

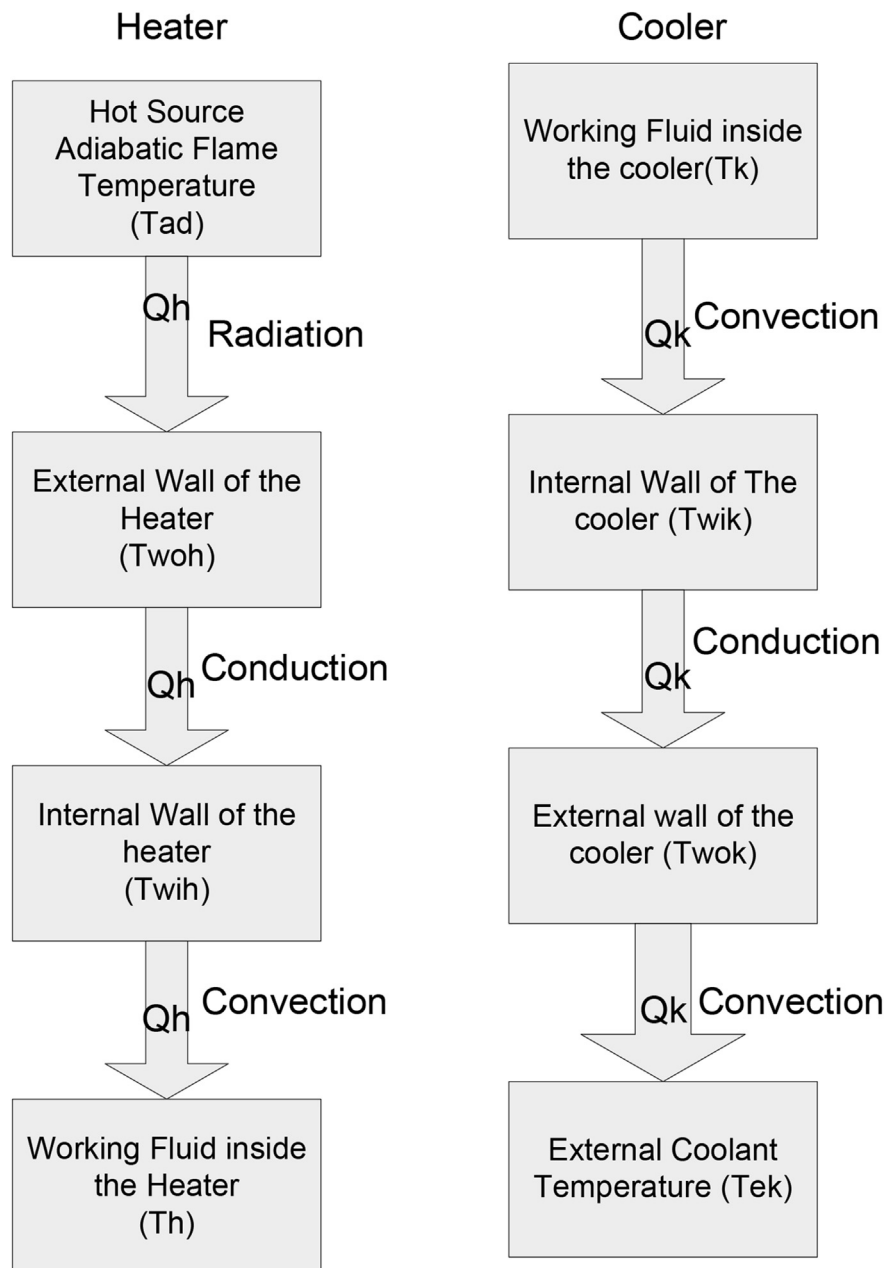


Fig. 3. Heat transfer model.

The conduction heat transfer through the wall follows the Fourier law of conduction and the convection heat transfer rate is consistent with Newton's cooling Law. This is expressed in terms of thermal resistances. Thus, considering an additional fouling resistance for the internal wall of the tubes (R_{fih}), the heat flux is expressed with the following equation.

$$Q_h = \frac{1}{R_{cih} + R_{hih} + R_{fih}} (T_{woh} - T_h) \quad (1)$$

Where, R_{cih} , R_{hih} , R_{fih} are the conductive, convective and fouling thermal resistances respectively. The conductive resistance depends mainly on the heat exchangers thermal properties. On the other hand the convective resistance mainly depends on the flow characteristics which are directly related to the convection coefficient (h_{ih}). Therefore considering that the working gas is pressurized at high density and moving with high velocity, this coefficient is estimated with the following expression derived from the Reynolds analogy [35].

$$h_{ih} = \frac{C_{fh} \cdot C_{ph} \cdot \mu_h}{2d_{ih} Pr_h} \quad (2)$$

where C_f is the friction factor, which is calculated with Blasius Correlations [35]. It is important to mention that the transport properties depends on the working gas temperature at the heater (T_h), which is assumed to be constant along the heat exchanger

2.2.1.2. Cooler. The heat transfer goes from the working gas inside the cooler space to the cooler outer wall. This assumes the following mechanism: Convective heat transfer from the working gas to the inner wall surface, and conductive heat transfer from the inner to the outer wall surface. Considering that the assumptions are analogous to the heater module, the heat flux in the cooler space is estimated with a similar expression:

$$Q_k = \frac{1}{R_{cik} + R_{hik} + R_{fik}} (T_{wik} - T_k) \quad (3)$$

The gas flow is also pressurized and moving with high velocity inside the cooler, therefore the internal heat transfer coefficient is also estimated with the Reynolds analogy.

$$h_{ik} = \frac{C_{fk} \cdot C_{pk} \cdot \mu_k}{2d_{ik} Pr_k} \quad (4)$$

2.2.1.3. Regenerator. The regenerator is a simplified thermal model that considers the geometrical characteristics, matrix packing, matrix porosity, and the influence of different materials to evaluate the non-ideal regeneration. It is assumed that a unidirectional and incompressible working fluid flows from the heater to the cooler at crank angles corresponding to 0–180°, and from the cooler to the heater at angles 180–360°. During the first blow (0–180), the heat is rejected from the hot working fluid to the regenerator material and during the reverse flow the heat is transmitted back to the working fluid. Considering these assumptions, the imperfect regeneration is evaluated in terms of the regenerator effectiveness (ϵ), which is defined according to the enthalpy change [12]. Based on this definition, the heat loss from the regenerator is determined by:

$$Q_{lossr} = (1 - \epsilon) \times Q_r \quad (5)$$

The regenerator effectiveness could also be expressed considering the following definition:

$$\epsilon = \frac{NTU}{1 + NTU} \quad (6)$$

NTU is the number of transfer units, which is related with the Stanton Number (St), and the wetted area in the regenerator [38].

$$NTU = \frac{St}{2} \left(\frac{A_{wg}}{A} \right) \quad (7)$$

The Stanton number is calculated from Nusselt number correlations evaluated by Thomas and Pittman [36]. From these, the wire screens, and metal felts correlations are shown in equations (8) and (9) respectively.

$$Nu = (1.010 + 0.503Re^{0.720}) \times ((1 - 0.847) \times (1 - \epsilon)) \quad (8)$$

$$Nu = (0.963 + 0.352Re^{0.794}) \times ((1 - 1.485) \times (1 - \epsilon)) \quad (9)$$

2.2.2. External heat transfer module

The external heat transfer module considers the heat flow from the hot or cold source to the external walls of the heat exchangers. This module permits the integration of the engine with different heat sources. In this case the integration with an external combustion chamber and a water cooling system is considered.

2.2.2.1. Heater. This module considers the heat flow from the flame in the combustion chamber to the heater external wall. The source is supposed to reach the adiabatic flame temperature, and therefore thermal radiation is assumed as the main heat transfer mechanism. Considering these assumptions, the heat flow (Q_h), is calculated from Boltzman's Law [37]. This is reordered in terms of thermal resistances, including an additional fouling resistance as shown in equation (10).

$$Q_h = \frac{1}{\frac{1}{h_{rh} A_{oh}} + R_{foh}} (T_{ad} - T_{woh}) \quad (10)$$

The term h_{rh} is the radiation heat transfer coefficient, which is estimated with the following equation [35]:

$$h_{rh} = \alpha \sigma F_R (T_{ad} + T_{woh}) \quad (11)$$

The fouling factor strongly depends on the fuel characteristics. For example in the case of solid bio fuels, which give high amount of ash residues, this factor R_{foh} varies from 0 to 40 m²K/kW [39].

2.2.2.2. Cooler. The external heat transfer model at the cooler side, considers the heat flow from the outer surface of the cooler to the external cooling fluid. This fluid flows outside of the heat exchanger in a cross flow configuration and it is assumed that the heat transfer is mainly due to a convective mechanism. Additionally since water cooling is typically used on these systems, this model includes water as the main cooling fluid. However, other fluids could be easily implemented into the model.

The heat flow (Q_k) and the external wall temperature of the heat exchanger are therefore estimated combining Newton's cooling Law for convective heat transfer and the energy balance. Combining these equations, the following expression to estimate the external wall temperature on the cooler outer wall is obtained.

$$T_{w ok} = T_{water_in} + Q_k \left(\frac{1}{h_{ok} A_{ok}} + \frac{1}{2m_{water} C_{pwater}} \right) \quad (12)$$

The external heat transfer coefficient (h_{ok}), is estimated using Coulburn correlation for external flow in a bank of tubes

[35]. Additionally a Fouling factor is assumed on the water side. Therefore the total heat transfer coefficient is expressed as:

$$h_{ok} = \frac{1}{\frac{1}{h_{water}} + R_{fok} A_{ok}} \quad (13)$$

2.3. Energy losses

The energy losses due pressure drop and thermal losses are evaluated on this module. These are assumed independent and calculated after reaching cyclic steady-state conditions. This complements the losses due dead volumes, limited heat transfer and imperfect regeneration considered in previous modules.

2.3.1. Pressure drop in the heat exchangers

Tew et al. [20], showed that combining the continuity and momentum equation for one dimensional flow, and considering that the Stirling engine reach cyclic steady-state conditions the following expression, that evaluates the pressure drop over a finite length, is obtained.

$$\Delta P = \frac{f}{d_{hy}} \frac{1}{2} \rho v^2 L \quad (14)$$

The friction factor coefficient (f) is estimated using steady flow correlations for flow inside tubes [35].

2.3.2. Pressure drop in the regenerator

The regenerator pressure drop has a significant effect on both efficiency and power output of the Stirling engines. According to Chen and Griffin [40], the pressure drop in the regenerator accounts for 70–90% of the total pressure drop through the heat transfer components. In this sense updated correlations, evaluated by Thomas and Pitt [36], which consider oscillating flow conditions are used in this module. Furthermore these correlations include a wide range of parameters such as different Reynolds numbers, Valensi numbers, flush ratio, wire mesh width, wire diameter and porosity.

The following expression evaluates the pressure drop [26]:

$$\Delta P = C_f n \frac{\rho}{2} u^2 \quad (15)$$

The pressure drop is proportional to the number of flow resistors (n), the square of the mean velocity (u^2) and the friction coefficient (C_f). Additionally the friction coefficient considers the form drag (C_{fd}) and skin friction (C_{sf}), with the following expression:

$$C_f = C_{fd} + \frac{C_{sf}}{Re} \quad (16)$$

Where,

$$\begin{aligned} C_{sf} &= 68.556 & C_{fd} &= 0.5274 & \text{wire screens} \\ C_{sf} &= 70.035 & C_{fd} &= 0.9307 & \text{metal felts} \end{aligned}$$

2.3.3. Pumping loss

The pumping loss per cycle is calculated by integration of the product of the total pressure drop on the heat exchangers and the derivative of the expansion volume [38].

$$W_{ploss} = \int_0^{2\pi} \left(\sum_{i=1}^3 \Delta P_i \times \frac{dV_e}{d\theta} \right) \cdot d\theta \quad (17)$$

2.3.4. Energy losses due to internal conduction

The temperature difference along the engine promotes an internal conduction through the walls of the different components. Among these, it is important to consider the losses from heater to the cooler, due to the internal conduction through the walls of the regenerator [38]. These are estimated from:

$$Q_{lk} = \frac{k_r A_r}{L_r} (T_{wih} - T_{wik}) \quad (18)$$

Where, Q_{lk} , k_r , L_r , and A_r denote the heat loss, regenerator matrix conductivity, regenerator length, and regenerator conductive area respectively.

2.3.5. Energy losses due to shuttle conduction

Shuttle conduction occurs due to the oscillation of the hot displacer across a temperature gradient. The displacer absorbs the heat during the hot end of its stroke and then released it during the cold end of its stroke. This loss is estimated with the following expression [41].

$$Q_{lsh} = 0.4 \frac{Z^2 k_{pist} D_d}{J L_d} (T_e - T_c) \quad (19)$$

Where, Z is the displacer stroke, K_{pist} the piston thermal conductivity, D_d the displacer diameter, and J the annular gap between the displacer and the cylinder

2.4. Brake work and heat requirements

The brake work is evaluated by subtracting the pumping (W_{ploss}) and mechanical losses (W_{mech}) to the net work calculated with the ideal adiabatic model.

$$W_{br} = \oint (P dV_e + P dV_c) - W_{ploss} - W_{mech} \quad (20)$$

The mechanical losses are determined experimentally for the particular engine, through correlations that relates these losses to the engine frequency [42].

The total heat requirements are also calculated including the thermal losses previously evaluated. It is assumed that these losses are restored by the heat exchangers, increasing the total heating (Q_{ht}), and cooling requirements (Q_{kt}):

$$Q_{ht} = Q_h + Q_{lossr} + Q_{lk} + Q_{lsh} \quad Q_r \geq 0 \quad (21)$$

$$Q_{kt} = Q_k + (1 - \epsilon) \cdot Q_r \quad Q_r < 0 \quad (22)$$

3. Solution

3.1. Model layout

The different modules that compose the Stirling model require different operational and geometrical inputs. These are arranged in model blocks that were programmed with ACM language. Their interrelation is shown in Fig. 4. From this figure, it is seen that the heater cooler and regenerator blocks, which contain the heat exchangers geometrical variables; and the Comp-Exp block, that contains the engine geometrical and operational variables; are directly connected to the main block called Stirling. This block contains the main routine, including the C++ linkage for the model solution, and it also requires the information from the external heat source, cooling fluid and working gas properties blocks. The table

presented in Appendix B, describes in greater detail the mentioned blocks. Fig. 5

3.2. Numerical solution

The set of algebraic differential equations for the engine and the heat transfer modules is solved using an iterative initial value numerical method. The set of equations is solved at each time step using a fourth order Runge Kutta scheme for the time discretization. This is repeated until cyclic steady-state conditions, which is reached when the difference between the assumed initial values and the values calculated at the end of the cycle are lower than a defined error.

The numerical solution was implemented in a C++ procedure that is coupled to the ACM model. The following scheme describes the iterative steps for the solution. Fig. 5

4. Validation of the model

The GPU-3 Stirling engine [33] was chosen for the simulation and model validation. This is a single cylinder displacer engine, with rhombic and sliding rod seals. It is capable to produce approximately 7.5 kW with hydrogen working fluid at 6.9 MPa and 3600 rpm rotational speed [33]. This engine was studied by NASA-Lewis Research Centre (LeRC) [33], and the geometrical dimensions

and operational characteristics are well documented. In addition, the NASA-Lewis Research Centre developed a computational model [20], which was also compared with the ACM model predictions.

4.1. Brake power comparison with experimental reports

The engine brake power is defined as the net brake work per cycle times the engine frequency (freq).

$$P_{br} = W_{br} \times \text{freq} \quad (23)$$

The ACM model calculates the brake power considering the mechanical losses measured experimentally by LeRC [43]. Table 1, presents the brake power predicted by the model at different frequencies, considering hydrogen as working fluid at a mean pressure equal to 2.76 MPa, and the heater temperature (T_h) equal to 704 K. These results are compared with LeRC experimental and numerical data. In addition, the results for the simulation at three different mean pressures ($P = 1.38$ MPa; $P = 2.76$ MPa; $P = 4.14$ MPa) are shown in Fig. 6.

The previous results showed the capability of the model under different pressure levels. These results are complemented with the brake power calculation considering different temperatures on the hot side of the engine. This is shown in Fig. 7.

Table 1, and Figs. 6 and 7 show that both the ACM and LeRC models present similar degree of accuracy when comparing with

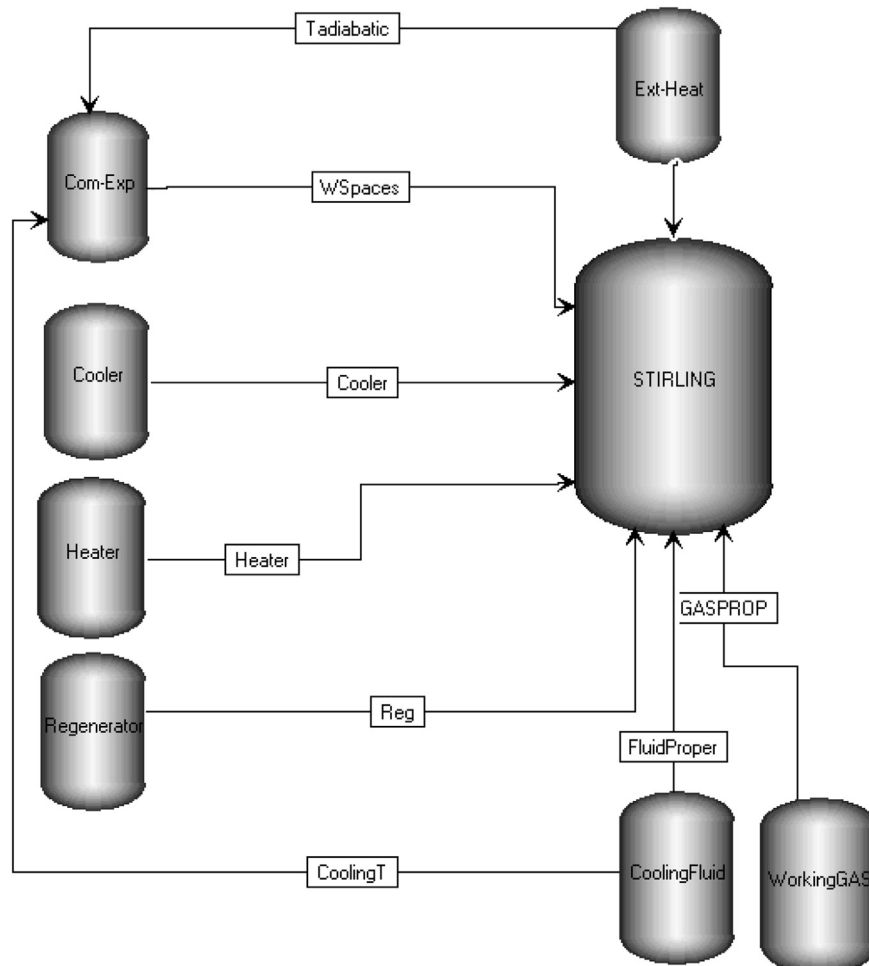


Fig. 4. Stirling engine model ACM layout.

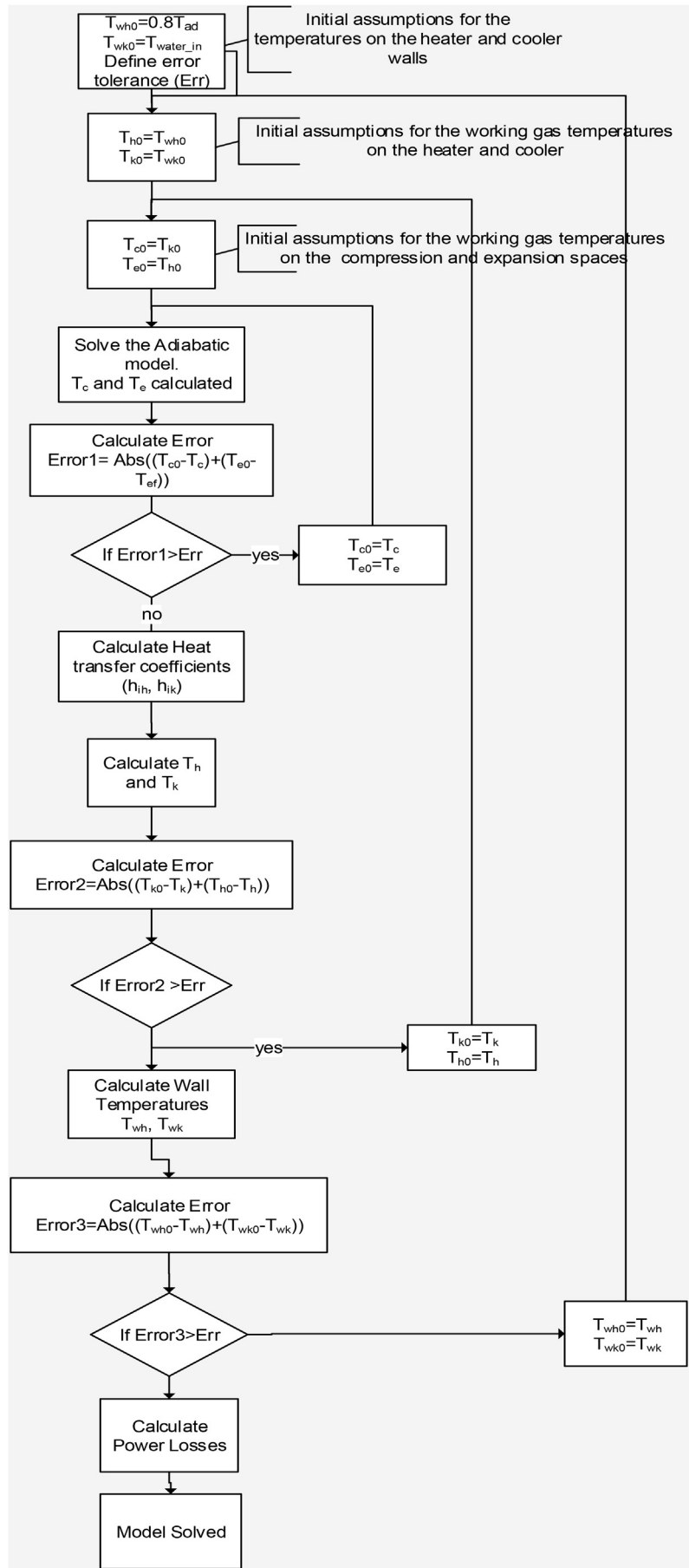


Fig. 5. Numerical solution for the Stirling engine system.

Table 1

Estimated and predicted brake power for the GPU3 engine.

Frequency (Hz)	Estimated indicated power (kW)	Estimated pumping losses (kW)	Mechanical losses (kW)	Brake power ACM (kW)	Brake power LeRC (kW)	Measured brake power (kW)	Absolute error ACM (%)	Absolute error LeRC (%)
16.67	1.60	0.06	0.48	1.06	1.14	1.13	6.15	0.9
25	2.19	0.14	0.58	1.48	1.60	1.49	0.84	7.38
33.33	2.90	0.26	0.67	1.96	2.13	1.95	0.84	9.23
41.67	3.57	0.43	0.77	2.37	2.50	2.39	0.68	4.61
50	4.24	0.65	0.88	2.73	2.75	2.61	4.83	5.36
58.33	4.89	0.92	0.98	2.93	2.80	2.70	8.40	3.70

the experimental measures at different frequencies, pressure levels and heater temperatures. For the higher temperature ($T_h = 704\text{ }^\circ\text{C}$) and intermediate pressure level, the ACM model presents more accurate results than the LeRC model, with a maximum error around $\pm 5\%$ for middle frequencies. However, it tends to over predict the brake power at higher frequencies, within an error around $\pm 8\%$. This pattern is similar at the low pressure operation, where the ACM model still presents better accuracy at lower frequencies, but slightly over predicts the brake power at higher frequencies. In the case of the high pressure operation, only one experimental point was reported [43], and for this point both the ACM and LeRC models present similar accuracy. The brake power prediction at lower temperatures is also similar for both models. However, the LeRC model presents slightly better results at this temperature.

In general the ACM and LeRC models are limited by the underestimation of the pumping losses through the engine. The use of correlations for oscillating flow conditions on the ACM model pretended to obtain better estimates of these losses, which is reflected on the accuracy at middle frequencies. On the other hand the use of several control volumes on the LeRC model captures better the variation of the gas properties through the engine, and thus the results at lower temperatures and higher frequencies were more accurate. In addition, the experimental report pointed out that the underestimation of the losses may be also due to oil contamination on the regenerator and methane production by thermal cracking of this contaminant. This may

explain the higher differences at higher temperatures and frequencies. Therefore, the ACM model capability for the prediction of the brake power is good enough for the engine analysis. Furthermore, the second order approach requires less computing resources than the LeRC approach, thanks to the use of less number of control volumes, and thus makes the ACM model suitable for larger studies such as overall optimizations.

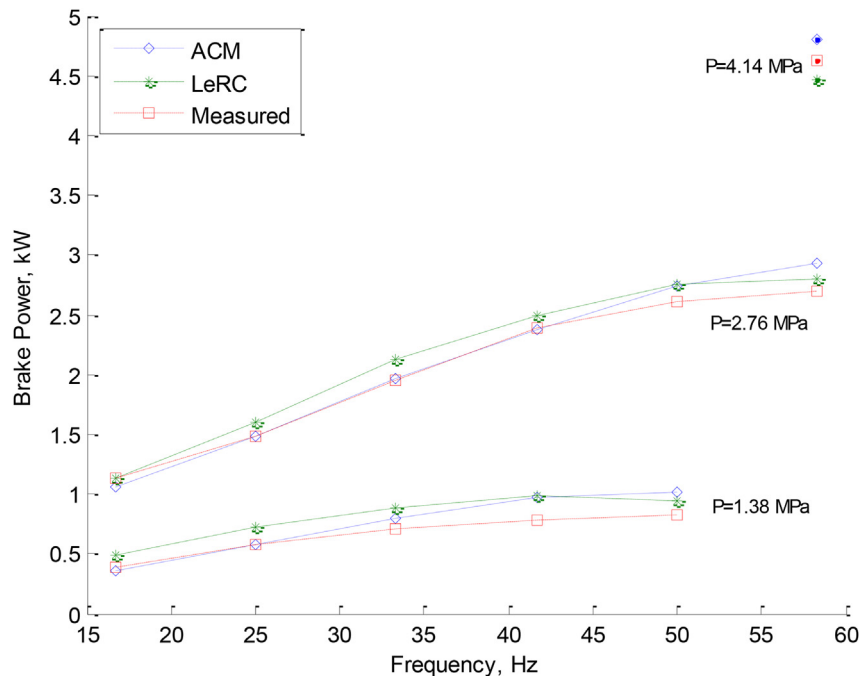
4.2. Efficiency

The engine brake efficiency is defined as the brake power divided by the heat flow to the engine.

$$\eta_b = \frac{P_{br}}{Q_{ht}} \quad (24)$$

Table 2 compares the brake power predicted by the ACM and LeRC models at different frequencies, considering hydrogen as working fluid at a mean pressure equal to 2.76 MPa, and the heater temperature equal to $704\text{ }^\circ\text{C}$. These are compared with the experimental results reported by the LeRC GPU3 tests [43]. In addition, Fig. 8 illustrates the variation of the engine efficiency at different pressure levels.

The results reflect the difficulty to predict the brake efficiency for the system. This is due to the complexity to evaluate the real heat transfer into the engine. It is important to mention that the

**Fig. 6.** Brake power $T_h = 704\text{ }^\circ\text{C}$; $T_{water,in} = 15\text{ }^\circ\text{C}$.

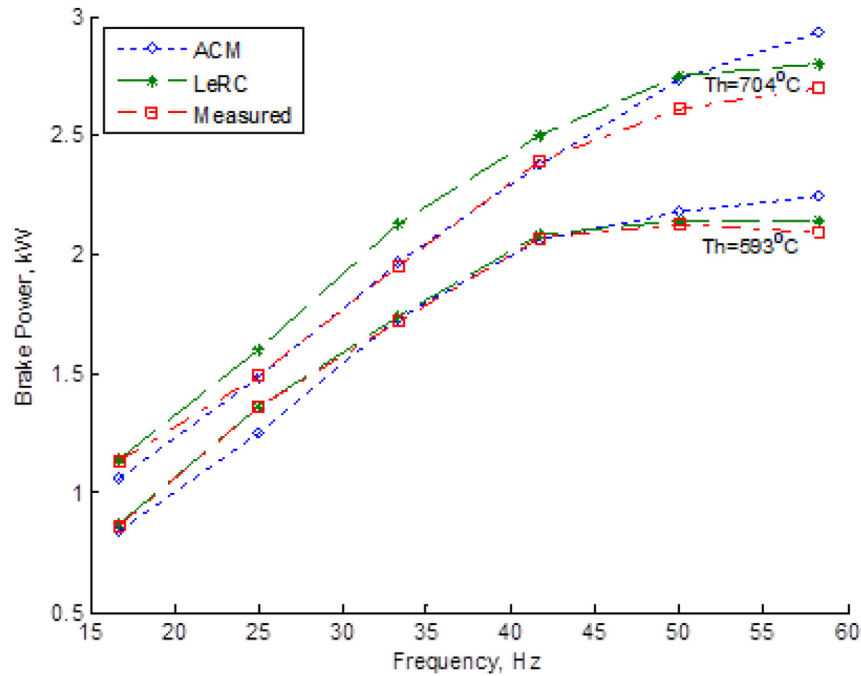


Fig. 7. Brake power at mean pressure = 2.76 MPa; $T_{\text{water, in}} = 15^\circ\text{C}$.

experimental values for the heat into the engine were estimated considering a hot energy balance and a complete combustion of the fuel. Therefore the experimental results have an intrinsic error reflected on some spread values. Despite this difficulty, the ACM model presents acceptable predictions at the different pressure levels. The errors on the estimation are around $\pm 8\%$ for frequencies higher than 16.67 Hz, with a maximum of $\pm 14\%$ at this frequency. In addition, as seen on Fig. 8, these results are more accurate than the LeRC results which overestimate the predictions at the different pressure levels. The better capability that the ACM presents to predict the brake efficiency may be explained with the use of correlations suitable for oscillating flow conditions, for the heat transfer through the regenerator [43]. From these comparisons, it can be concluded that the capability for brake efficiency predictions that the ACM model presents is valid and adequate for the design and optimization studies that aims to assist.

5. Results and discussion

The previous section analysed the model capacity to predict the brake power and efficiency of the engine. However, a broader analysis of the engine could be addressed analysing additional results that the model predicts. Therefore, this section presents additional outcomes for the simulation of the GPU-3 Stirling engine, considering the operational conditions shown in Table 3.

Table 2
Comparisons for the engine brake efficiency.

Frequency (Hz)	Brake efficiency ACM (%)	Brake efficiency LeRC (%)	Measured brake efficiency (%)	Error ACM (%)	Error LeRC (%)
16.67	20.8	28.09	24.10	13.69	16.55
25	23.24	29.81	23.45	0.89	27.12
33.33	23.93	30.71	25.82	7.32	18.94
41.67	24.49	29.83	25.64	4.49	16.34
50	24.98	28.00	25.68	2.73	9.03
58.33	24.30	25.00	23.06	5.38	0.24

Furthermore, some of these results are compared with the LeRC model calculations.

5.1. Volume variation

Fig. 9, shows the ACM model and LeRC results for the volume variation in the expansion and compression spaces during a complete cycle. The curves are sine waves corresponding to the dynamics of the piston and displacer connected through a rhombic drive crank mechanism. These variations were determined from the geometric relations among the parts of the mechanism [20]. The mechanism provides a symmetric motion between both piston and displacer for this configuration.

5.2. Pressure variation

It was assumed that the pressure is uniform along the engine but it has a transient variation during the cycle. These variations are shown in Fig. 10 and from these a slight difference between the ACM and LeRC results are noticed. The difference is explained because the LeRC model includes a coupled correction for the pressures losses along the heat exchangers. On the other hand, the ACM model considers decoupled pressure drops, and thus these are subtracted after the steady-state condition is reached. Despite of these differences, both approaches computed a similar sinusoidal variation during the cycle but the ACM approach requires less computing effort.

Fig. 11 shows the variation of the volumes inside the expansion and compression spaces and the pressure during the cycle. These pressure–volume plots were computed with the ACM model and aimed to reflect the behaviour of each working space. The integral of each curve correspond to the expansion and compression work shown in Table 4. From these curves it can be noticed that the net work may be increased with higher pressures during the expansion, but lower pressures for the compression processes. This will shrink the compression curve and enlarge the expansion one.

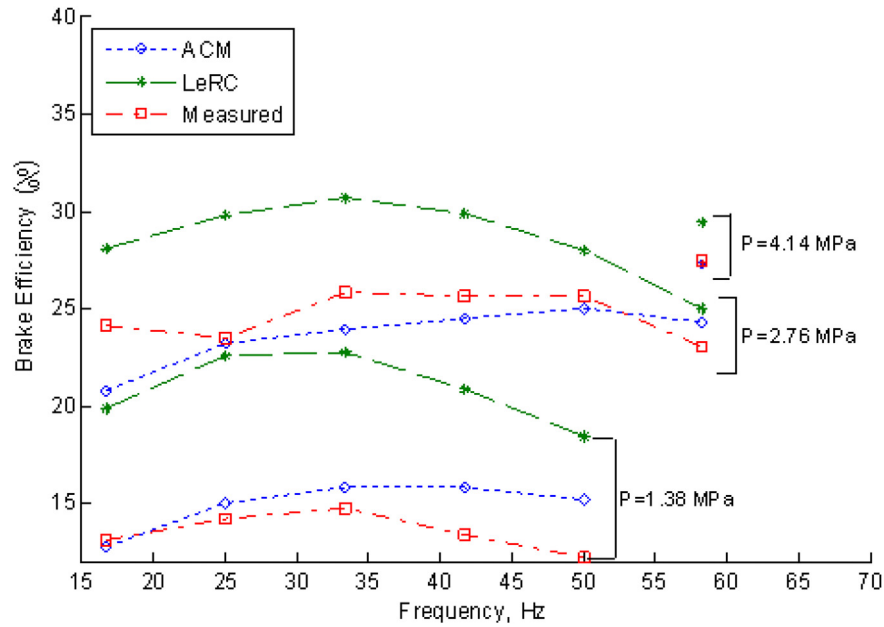


Fig. 8. Engine Efficiency at Mean Pressure = 2.76 MPa; $T_{\text{water,in}} = 15\text{ }^{\circ}\text{C}$.

Table 3
Ground power unit ACM simulation run.

Working fluid	Hydrogen
Frequency, Hz	50
Load pressure, N/m^2	5.5×10^6
Average heater temperature, K	976
Average cooler temperature, K	300

5.3. Mass distribution

The mass distribution during the cycle obtained with the ACM model, is shown in Fig. 12. This figure shows that during the first part of the cycle the mass in the compression space increases, while the mass in the expansion space starts to decrease. This is then

followed by a period, around $t = 0.008$ to 0.012 , where the changes are less pronounced and finally a phase where the mass in the expansion and compression spaces noticeably increase and decrease respectively.

The changes described correspond to the compression, heating-cooling, and expansion processes respectively and agree with the cyclic volume variations. In addition, the mass distribution through the other heat exchangers presents slight variations and thus reflects the validity of using the average steady-state correlation for their calculations. On the other hand, it is important to notice that the mass on the regenerator space is lower than in the other spaces. This means a low pressure drop but also a poorer heat transfer rate. Therefore, the effect of increasing the size of the regenerator on the engine performance should be further studied.

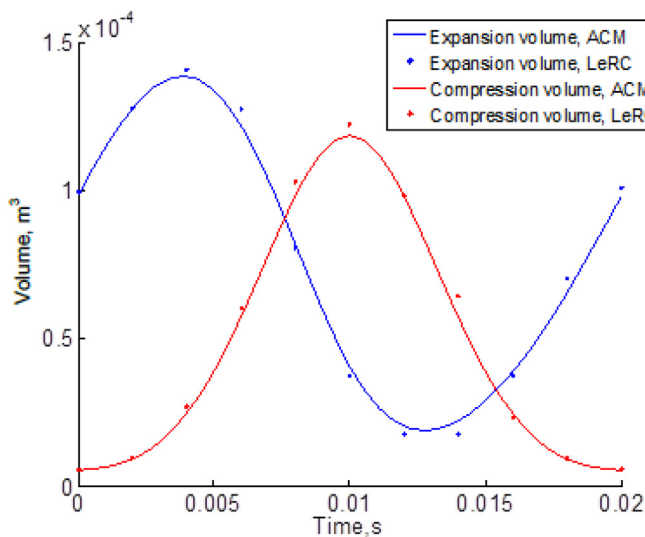


Fig. 9. Expansion-compression space volumes.

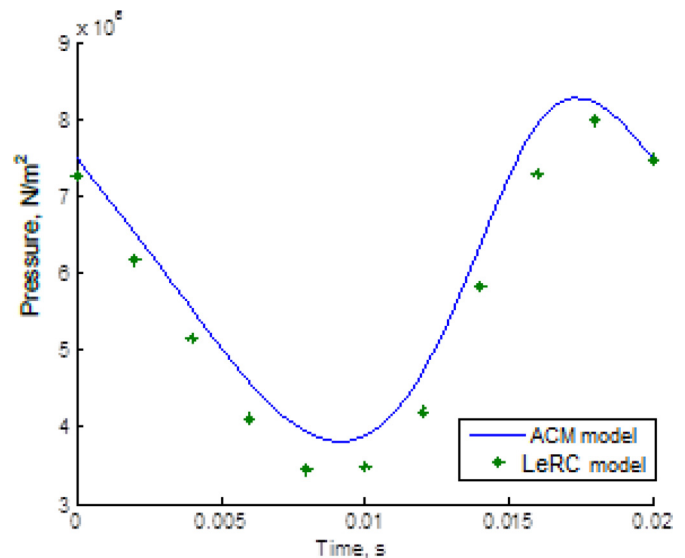


Fig. 10. Pressure variation as function of time.

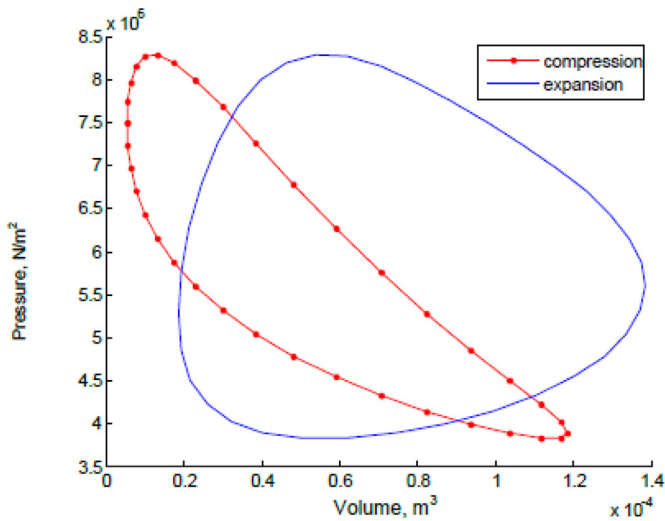


Fig. 11. ACM expansion and compression pressure–volume diagrams.

Table 4
Engine work per cycle.

Work per cycle	ACM
Expansion work (W_e , J/cycle)	395
Compression work (W_k , J/cycle)	170
Lost work due to pressure drop (J/cycle)	16
Net work (W_n , J/cycle)	209

5.4. Heat flow

Table 5, presents the results for the heat flow through the heater, cooler and regenerator at the end of a single cycle obtained with the ACM model. The heat flow in the regenerator is close to the expected ideal zero condition at the end of the cycle.

The cyclic variation for the heat flow is additionally shown in Fig. 13. The first half of the cycle is characterized by the increasing heat requirement on the heater, and the also increasing heat rejected to the regenerator. On the other hand, the last half of the cycle shows the heat rejected in the cooler, and the positive heat flow from the regenerator to the working fluid. These variations

Table 5
ACM model –Stirling Engine Heat Flow for cycle.

	Heat flow
Heater heat flow (Q_{hc} , J/cycle)	395
Cooler heat flow (Q_{nc} , J/cycle)	–170
Regenerator heat flow (Q_{rc} , J/cycle)	–0.04

correspond with the mass distribution and volume changes in the engine. Furthermore, it is important to notice the key importance of the regenerator, since it handles large part of the heat flow in the system.

The ACM model also includes the calculation of the losses due to: non-ideal regeneration, conductive and shuttle effect losses. The variation of these losses at different engine frequencies and the corresponding total heat requirements are shown in Table 6.

As seen on the table above, the changes on the internal and shuttle conduction losses with the operational frequency are almost negligible. But on the other hand, the losses due to imperfect regeneration increases largely and thus becomes the most important lost at higher frequencies. It is important to consider this when operating at high frequencies, since the engine performance could be seriously affected with an inefficient regeneration process.

5.5. Temperature variation

The temperature inputs and fouling factors assumed for the calculations are shown in Table 7.

Considering these inputs, the temperatures at the heat exchangers walls were estimated. This allowed to obtain the distribution shown in Fig. 14, which shows considerable differences between the temperature at the heat exchangers walls (T_{wk}, T_{wh}), and the temperatures of the working fluid at the different engine spaces (T_k, T_h, T_c, T_e). This reflects the correction for limited heat transfer included through the correlations in the model. Consequently, the engine efficiency computed with the model will tend to be lower than ideal second order approaches. In addition, including the fouling on the heat exchangers would further reduce this efficiency. However, the fouling factor should be carefully evaluated considering the particularities of the case studied.

6. Conclusions

In this paper, a Stirling engine model was developed using a modular methodology. This model included four modules, the

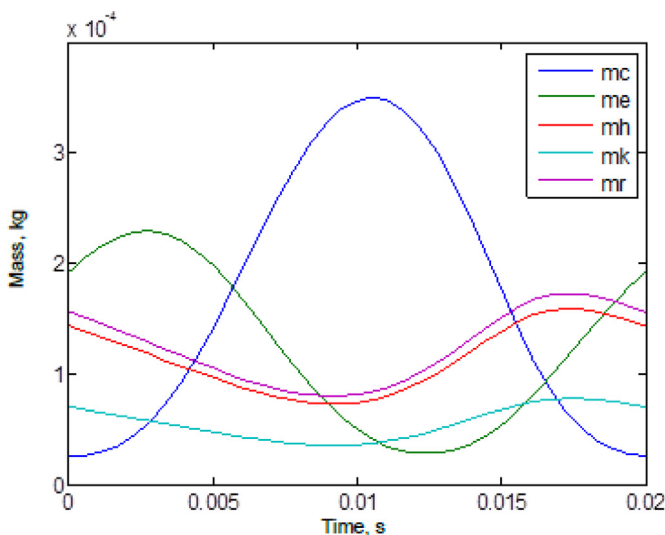


Fig. 12. Mass distribution along the engine obtained with the ACM model.

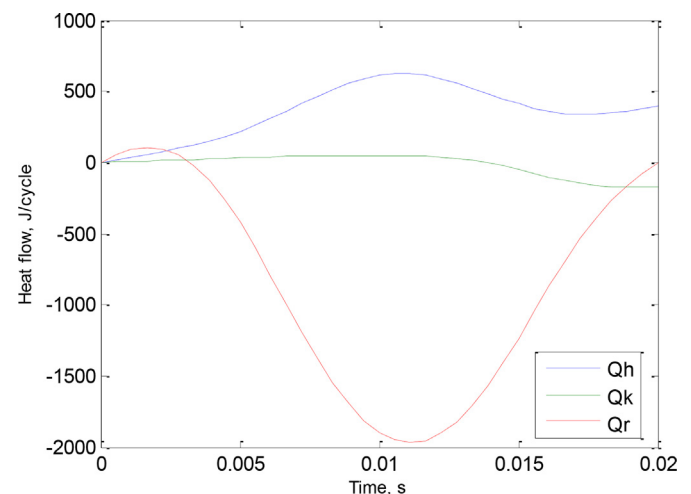


Fig. 13. Heat flow through the engine obtained with the ACM model.

Table 6

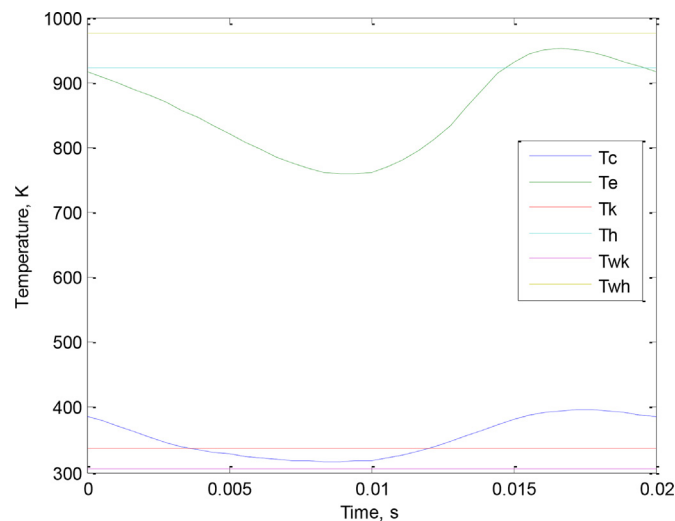
Thermal losses and requirements for the GPU3 engine obtained with the ACM model.

Frequency (Hz)	Heater flow (Q_h , kW)	Internal conduction losses (Q_{ik} , kW)	Shuttle conduction losses (Q_{ish} , kW)	Regenerator losses (Q_{lossr} , kW)	Total heat requirement (Q_{ht} , kW)
16.67	2.98	0.47	0.62	0.14	4.21
25.00	4.48	0.47	0.62	0.26	5.83
33.33	5.95	0.47	0.62	0.38	7.42
41.67	7.40	0.48	0.60	0.52	9.00
50.00	8.85	0.48	0.61	0.66	10.60
58.33	10.28	0.48	0.60	0.80	12.16

Table 7

Temperature and fouling factors.

T_{ad}	1410 K
T_{water_in}	298 K
R_{foh}	0 K/W
R_{fok}	0 K/W

**Fig. 14.** Temperature variation along the engine.

internal and external heat transfer modules, the ideal adiabatic module and the energy losses module. The proposed external heat transfer module and the modular methodology permitted a coupling of the Stirling engine governing equations and the heat transfer

phenomena on the system. This novel approach allows a smooth integration of the engine model with combined heat and power systems and thus will allow overall design-optimization studies. The model also included updated heat transfer and pressure drop correlations for the engine heat exchangers, with a special focus on the regenerator that considered correlations for cyclic flow conditions.

The proposed model was validated with experimental and numerical data reported for the GPU-3 Stirling engine. The results of the validation showed that the model present a good capability to compute the power output and engine efficiency under different frequencies, pressures and hot side temperatures. Furthermore, additional results for the variations of the volume, pressure, temperature, mass and heat flow, and energy losses through the cycle provided a deeper insight into the engine operation.

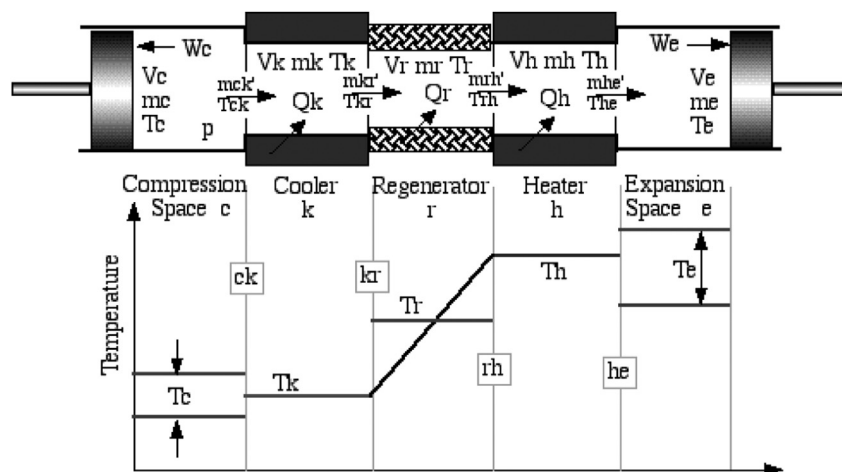
Therefore, despite of some limitations the model represents an adequate fast tool, useful for first design and subsequent optimization of the Stirling engine, with a capability for the integration into overall systems. In this sense, future studies will focus on the implementation of this modular model for the design analysis of overall Stirling engine based CHP plants.

Acknowledgements

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Appendix A

The scheme for the ideal adiabatic model, developed by Urielli, and its corresponding set of equations are shown below.

**Figure A 1.** Stirling engine adiabatic model scheme.

Mean pressure and pressure variation

$$P = \frac{MR}{\left(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e}\right)}$$

$$dP = \frac{-\gamma P \left(\frac{dV_c}{T_{ck}} + \frac{dV_e}{T_{he}} \right)}{\left[\frac{V_c}{T_{ck}} + \gamma \left(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} \right) + \frac{V_e}{T_{he}} \right]}$$

Mass of the working gas

$$m_c = p \left(\frac{V_c}{RT_c} \right); m_k = p \left(\frac{V_k}{RT_k} \right); m_r = p \left(\frac{V_r}{RT_r} \right); m_h = p \left(\frac{V_h}{RT_h} \right); m_e = p \left(\frac{V_e}{RT_e} \right)$$

Mass accumulation

$$dm_k = m_k \left(\frac{dP}{P} \right) d\theta; dm_r = m_r \left(\frac{dP}{P} \right) d\theta; dm_h = m_h \left(\frac{dP}{P} \right) d\theta$$

$$dm_c = \frac{PdV_c + \frac{V_c dP}{\gamma}}{RT_{ck}}; dme = \frac{PdV_e + \frac{V_e dP}{\gamma}}{RT_{he}}$$

Mass flow

$$m_{ck} = -dm_c$$

$$m_{he} = dm_e \quad (A.2)$$

$$m_{kr} = m_{ck} - dm_k \quad (A.3)$$

$$m_{rh} = m_{he} + dm_h \quad (A.4)$$

Conditional temperatures

$$\text{If } m_{ck} > 0 \text{ then } T_{ck} = T_c \text{ else } T_{ck} = T_k$$

$$\text{If } m_{he} > 0 \text{ then } T_{he} = T_h \text{ else } T_{he} = T_e$$

Temperatures

$$dT_c = T_c \left(\frac{dP}{P} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c} \right); dT_e = T_e \left(\frac{dP}{P} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e} \right)$$

Energy

$$dQ_k = \frac{V_k dP_{c_v}}{R} - c_p \left(T_{ck} m_{ck} - T_{kr} m_{kr} \right);$$

$$dQ_r = \frac{V_r dP_{c_v}}{R} - c_p \left(T_{kr} m_{kr} - T_{rh} m_{rh} \right)$$

$$dQ_h = \frac{V_h dP_{c_v}}{R} - c_p \left(T_{rh} m_{rh} - T_{he} m_{he} \right);$$

$$dW_c = PdV_c; dW_e = PdV_e$$

Appendix B*ACM- blocks description*

(A.1)

Block name	Main inputs	Main outputs	Description
Cooler–Heater	di: tubes internal diameter de: tubes external diameter len: Tubes length num: Number of tubes in bundle sl: Longitudinal space between tubes kw: wall thermal conductivity	a: cross sectional area of one tube ae: external wetted area aw: internal wetted area amin: minimal free flow area vk: cooler void volume	Geometrical characteristics for the cooler and heater heat exchangers
Regenerator	kindr: Regenerator Type wr: Open mesh width dwire: Wire diameter lr: Regenerator length por: Porosity	dhy: hydraulic diameter air: regenerator internal area vr: volume regenerator	Geometrical characteristic for the regenerator heat exchanger
Com-Exp	Alpha: phase angle between displacer and piston ccl: compression clearance cbor: cylinder bore crod: connecting rod length crnkr: crank radius drod: displacer rod diameter dd: displacer diameter dstr: displacer stroke ecl: expansion clearance ecc: Eccentricity prod: piston rod diameter pd: piston diameter	Vclc: compression space clearance volume Vcle: expansion space clearance volume Vswc: compression space swept volume Vswc: expansion space swept volume	Compression and Expansion Working Spaces characteristics as function of the drive mechanism and the engine configuration (alpha, beta, gamma)

(continued on next page)

(continued)

Block name	Main inputs	Main outputs	Description
Ext-Heat	tad : Adiabatic flame temperature during fuel stoichiometric combustion rfch: External fouling factor on the heater eair: Excess air on the combustion	tad: Adiabatic temperature for the combustion	Main characteristics for the external heat transfer. In this case the combustion process is considered
Cooling-Fluid	twik: Cooling fluid inlet temperature mwik: Cooling fluid mass flow fek: external fouling factor in the cooler	twok: Cooling fluid outlet temperature	Main Characteristic for the cooling external fluid. In this case the water is assumed
Working GAS	typeg: working fluid used (Air, Helium, hydrogen, Nitrogen)	Rgas: universal gas constant Cv: constant volume specific heat Cp: constant pressure specific heat Gamma: adiabatic constant Uo: Sutherland parameter To: Sutherland parameter So: Sutherland parameter	Physical properties for the working gas
Stirling	Pmean: mean pressure Freq: frequency Additionally all the outputs from previous modules	Th, Tk, Tr, Tc, Te dph, dpk, dpr Qh, Qk, Qr Wc, We, Wtot Eff: Engine efficiency	Main module calculate the different requirements and operational parameters for the Stirling engine

References

- [1] D.G. Thombare, S.K. Verma, Technological development in the Stirling cycle engines, *Renew. Sustain. Energy Rev.* 12 (2008) 1–38, <http://dx.doi.org/10.1016/j.rser.2006.07.001>.
- [2] T. Li, D. Tang, Z. Li, J. Du, T. Zhou, Y. Jia, Development and test of a Stirling engine driven by waste gases for the micro-CHP system, *Appl. Therm. Eng.* 33 (2012) 119–123.
- [3] J.J. Klemes, S. Pierucci, V. Kuhn, J. Klemes, I. Bulatov, MicroCHP: overview of selected technologies, products and field test results, *Appl. Therm. Eng.* 28 (2008) 2039–2048.
- [4] B. Thomas, Benchmark testing of Micro-CHP units, *Appl. Therm. Eng.* 28 (2008) 2049–2054, <http://dx.doi.org/10.1016/j.applthermaleng.2008.03.010>.
- [5] M. Dentice d'Accadia, M. Sasso, S. Sibilio, L. Vanoli, Micro-combined heat and power in residential and light commercial applications, *Appl. Therm. Eng.* 23 (2003) 1247–1259, [http://dx.doi.org/10.1016/S1359-4311\(03\)00030-9](http://dx.doi.org/10.1016/S1359-4311(03)00030-9).
- [6] I. González-Pino, A. Campos-Celador, E. Pérez-Iribarren, J. Terés-Zubiaga, J.M. Sala, Parametric study of the operational and economic feasibility of Stirling micro-cogeneration devices in Spain, *Appl. Therm. Eng.* (2013), ISSN: 1359-4311, <http://dx.doi.org/10.1016/j.applthermaleng.2013.12.020>.
- [7] M. De Paape, P. D'Herdt, D. Mertens, Micro-CHP systems for residential applications, *Energy Convers. Manag.* 47 (2006) 3435–3446, <http://dx.doi.org/10.1016/j.enconman.2005.12.024>.
- [8] R. Dyson, S. Wilson, R. Tew, Review of computational Stirling analysis methods, in: 2nd International Energy Conversion Engineering Conference, American Institute of Aeronautics and Astronautics, 2004, <http://dx.doi.org/10.2514/6.2004-5582>.
- [9] G. Walker, *Stirling-cycle Engines*, Clarendon Press, Oxford UK, 1973.
- [10] J.R. Senft, A simple derivation of the generalized Beale number, IECEC '82, in: Proceedings of the Seventeenth Intersociety Energy Conversion Engineering Conference, vol. 1, 1982, pp. 1652–1655.
- [11] F. Formosa, Coupled thermodynamic–dynamic semi-analytical model of free piston Stirling engines, *Energy Convers. Manag.* 52 (2011) 2098–2109.
- [12] C.-H. Cheng, Y.-J. Yu, Numerical model for predicting thermodynamic cycle and thermal efficiency of a beta-type Stirling engine with rhombic-drive mechanism, *Renew. Energy* 35 (2010) 2590–2601.
- [13] N. Seraj Mehdiadeh, P. Stouffs, Simulation of a Martini displacer free piston Stirling Engine for electric power generation, *Int. J. Thermodyn.* 3 (n.d.) 27–34.
- [14] I. Urieli, D.M. Berchowitz, *Stirling Cycle Engine Analysis*, A. Hilger, Michigan USA, 1984.
- [15] N. Parlak, A. Wagner, M. Elsner, H.S. Soyhan, Thermodynamic analysis of a gamma type Stirling engine in non-ideal adiabatic conditions, *Renew. Energy* 34 (2009) 266–273.
- [16] T. Finkelstein, Computer analysis of Stirling engines, *Adv. Cryog. Eng.* 20 (1975) (United States).
- [17] I. Urieli, C.J. Rallis, D.M. Berchowitz, Computer simulation of Stirling cycle machines, in: 12th Intersociety Energy Conversion Engineering Conference, vol. 1, 1977, pp. 1512–1521.
- [18] A. Schock, Nodal analysis of Stirling cycle devices, in: Intersociety Energy Conversion Engineering Conference, vol. 3, 1978, pp. 1771–1779.
- [19] D. Gedeon, Sage – object-oriented software for Stirling machine design, in: Intersociety Energy Conversion Engineering Conference, American Institute of Aeronautics and Astronautics, 1994.
- [20] R.C. Tew, K. Jefferies, D. Miao, U.S.D. of E.D. of T.E. Conservation, L.R. Center, *A Stirling Engine Computer Model for Performance Calculations* (Google eBook), Department of Energy, Office of Conservation and Solar Applications, Division of Transportation Energy Conservation, 1978.
- [21] A.J. Organ, Gas dynamics of the temperature-determined Stirling cycle, *Archive J. Mech. Eng. Sci.* 23 (1981) 207–216, 1959–1982 (vols 1–23).
- [22] V.H. Larson, Characteristic dynamic energy equations for Stirling cycle analysis, in: Proc., Intersoc. Energy Convers. Eng. Conf., vol. 2, 1981 (United States).
- [23] K. Mahkamov, D. Djumanov, Three-dimensional CFD modeling of a Stirling engine, in: Proceedings of the 11th International Stirling Engine Conference, vol. 19, 2003.
- [24] R.C. Tew, Zhang Zhiguo, Gedeon David, Terrence W. Simon, *CFD Modeling of Free-Piston Stirling Engines*, National Aeronautics and Space Administration, Glenn Research Center, 2001.
- [25] S. Wilson, R. Dyson, R. Tew, M. Ibrahim, Multi-D CFD modeling of a free-Piston Stirling Converter at NASA GRC, *Arc.aiaa.org*, (n.d.).
- [26] K. Alanne, N. Söderholm, K. Sirén, I. Beausoleil-Morrison, Techno-economic assessment and optimization of Stirling engine micro-cogeneration systems in residential buildings, *Energy Convers. Manag.* 51 (2010) 2635–2646.
- [27] K. Lombardi, V.I. Ugursal, I. Beausoleil-Morrison, Proposed improvements to a model for characterizing the electrical and thermal energy performance of Stirling engine micro-cogeneration devices based upon experimental observations, *Appl. Energy* 87 (2010) 3271–3282, <http://dx.doi.org/10.1016/j.apenergy.2010.04.017>.
- [28] H. Ribberink, D. Bourgeois, I. Beausoleil-Morrison, A plausible forecast of the energy and emissions performance of mature-technology Stirling engine residential cogeneration systems in Canada, *J. Build. Perform. Simul.* 2 (2009) 47–61.
- [29] V. Dorer, A. Weber, Energy and carbon emission footprint of micro-CHP systems in residential buildings of different energy demand levels, *J. Build. Perform. Simul.* 2 (2009) 31–46.
- [30] F. Miccio, On the integration between fluidized bed and Stirling engine for micro-generation, *Appl. Therm. Eng.* 52 (2013) 46–53.
- [31] I.B.-M. Nick Kelly, Specifications for modelling fuel cell and combustion-based residential cogeneration devices within whole-building simulation programs, in: Annex 42 of the International Energy Agency, October 2007.
- [32] A. Ferguson, N. Kelly, A. Weber, B. Griffith, Modelling residential-scale combustion-based cogeneration in building simulation, *J. Build. Perform. Simul.* 2 (2009) 1–14.
- [33] L.G. Thieme, U.S.D. of Energy, Division of Transportation Energy Conservation, L.R. Center, Low-power Baseline Test Results for the GPU 3 Stirling Engine, The Division, 1979.
- [34] Y.C. Hsieh, T.C. Hsu, J.S. Chiou, Integration of a free-piston Stirling engine and a moving grate incinerator, *Renew. Energy* 33 (2008) 48–54, <http://dx.doi.org/10.1016/j.renene.2007.01.015>.
- [35] F.P. Incropera, A.S. Lavine, D.P. DeWitt, *Fundamentals of Heat and Mass Transfer*, John Wiley & Sons, 2011.
- [36] B. Thomas, D. Pittman, Update on the evaluation of different correlations for the flow friction factor and heat transfer of Stirling engine regenerators, in: Collection of Technical Papers. 35th Intersociety Energy Conversion Engineering Conference and Exhibit (IECEC) (Cat. No.00CH37022), American Inst. Aeronaut. & Astronautics, 2000, pp. 76–84.
- [37] S.T. Hsu, F.Y. Lin, J.S. Chiou, Heat-transfer aspects of Stirling power generation using incinerator waste energy, *Renew. Energy* 28 (2003) 59–69.
- [38] J.M. Strauss, R.T. Dobson, Evaluation of a second order simulation for Stirling engine design and optimisation, *Energy South. Afr.* 21 (2010) 17–29.
- [39] M. Kuosa, J. Kaikko, L. Koskelainen, The impact of heat exchanger fouling on the optimum operation and maintenance of the Stirling engine, *Appl. Therm. Eng.* 27 (2007) 1671–1676, <http://dx.doi.org/10.1016/j.applthermaleng.2006.07.004>.

- [40] N.C.J. Chen, E.P. Griffin, Effects of pressure-drop correlations on Stirling engine predicted performance, in: *Proc., Intersoc. Energy Convers. Eng. Conf.; (United States)*, vol. 2, 1983.
- [41] Y. Timoumi, I. Tlili, S. Ben Nasrallah, Design and performance optimization of GPU-3 Stirling engines, *Energy* 33 (2008) 1100–1114, <http://dx.doi.org/10.1016/j.energy.2008.02.005>.
- [42] W.R. Martini, *Stirling Engine Design Manual*, University Press of the Pacific, USA, 2004.
- [43] R.C. Tew, *Initial Comparison of Single Cylinder Stirling Engine Computer Model Predictions with Test Results*, 1979 (DOE/NASA/1040-78/30), for sale by the National Technical Information Service, USA.